Integral Equation Results for the ^{4}He(e, e′p)^{3}H Reaction at High Missing Momenta

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(February 9, 2008)

The two-fragment electrodisintegration of ^{4}He into proton and triton is calculated in Plane Wave Impulse Approximation (PWIA). The three- and four-nucleon wave functions involved are obtained by solving the Alt-Grassberger-Sandhas (AGS) integral equations, with the Malfliet-Tjon potential as the underlying NN-interaction. Our results are in remarkable agreement with the experimental data and, in contrast to alternative approaches, do not exhibit any dip in the five-fold differential cross section at a missing momentum of $\sim 450\,\mathrm{MeV}/c$.

PACS numbers: 21.45.+v, 25.10.+s, 25.30.Fj

The two-fragment electrodisintegration process ^{4}He(e, e′p)^{3}H has been the subject of several experimental investigations for various kinematics (see, for example, [1–5]). On the theoretical side quite some effort has been devoted to calculating this process. However, the exact treatment of four-nucleon electrodisintegration observables is computationally very demanding and, thus, has usually been simplified by approximations and model assumptions [5–8].

In Plane-Wave Impulse Approximation (PWIA) all these calculations exhibit a characteristic dip, actually zero, in the five-fold differential cross-section around a missing momentum of $\sim 450\,\mathrm{MeV}/c$, which does not show up in the experimental data. Laget [7] performed calculations including final state interaction (FSI) effects and meson exchange currents (MEC) by means of a Feynman diagrammatic approach. Although this resulted in a partial filling of the dip, these investigations also underestimate the data considerably in this region. Similar results were obtained when FSI was taken into account via an effective nucleon-trinucleon interaction [6,8,9]. In a completely different approach Nagorny *et al.* [10] included the electromagnetic field within the strongly interacting system in a relativistic gauge invariant way, the FSI being incorporated via the pole contribution of the p^{3}H→ p^{3}H scattering matrix [11]. The agreement with the data is again fairly satisfactory, but the zero is exhibited as well [5].

In detail, at missing momenta less than $300\,\mathrm{MeV}/c$ all calculations show a good agreement with the data [2,5,8]. Surprisingly, the PWIA performs reasonably well in this region where the FSI could be expected to be more important than in the higher missing momenta region. In contrast, in the region $300\,\mathrm{MeV}/c < Q < 600\,\mathrm{MeV}/c$ the results strongly depend on the way the FSI effects are included. For example, as pointed out in [5] the Laget results underestimate the cross section by a factor of 4 and those of Schiavilla by a factor of 2. At missing momenta above $600\,\mathrm{MeV}/c$, where the MEC contribution is becoming important, the agreement with the data is again fair. We, therefore, conclude that the zero in the PWIA cross section is not necessarily a manifestation of strong FSI or MEC effects.

Instead, one should look for other explanations, such as the dependence of the results on the model used, the NN forces employed, the determination of the bound state wave functions etc. In the present work the wave functions involved were calculated within the exact three- and four-nucleon AGS formalism [12,13]. We mention that the same wave functions have successfully been employed already in calculations of the two-fragment photodisintegration of the α–particle [14,15].

We consider the two-fragment reaction in which the scattered electron and the ejected nucleon are measured in coincidence. The corresponding electron-proton coincidence cross section is given by

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}E_f d\Omega_p \mathrm{d}\Omega_e} = \frac{\sigma_\mathrm{M}}{(\hbar c)^3(2\pi)^3}\frac{\rho_f}{4E_iE_f\cos^2\dfrac{\theta}{2}}|\mathcal{M}(\mathbf{q})|^2 \quad (1)$$

where σ_M is the Mott differential cross section,

$$\sigma_\mathrm{M} = \frac{e^4\cos^2\dfrac{\theta}{2}}{4E_i^2\sin^4\dfrac{\theta}{2}}\,. \quad (2)$$

$E_i(E_f)$ is the energy of the incoming (outgoing) electron and ρ_f is the relativistic density of states. The transition matrix, properly antisymmetrized with respect to the four nucleons [16], is given by

$$\mathcal{M}(\mathbf{q}) = 2\,{}^{(-)}\langle \mathbf{q}; \Psi_{III}|H|\Psi_{IV}\rangle\,, \quad (3)$$

where H is the Hamiltonian describing the interaction between the electron and the nucleons. The ejected proton moves away with momentum $\mathbf{q}$ with respect to the residual three-nucleon bound state $|\Psi_{III}\rangle$. The kinematics of this process is shown in Fig. 1.

The Hamiltonian for the interaction between an electron and four nucleons is that of McVoy and van Hove [17], which has been previously employed in the electrodisintegration of the trinucleon system by Lehman and collaborators [18,19] and by Epp and Griffy [20]. This Hamiltonian, correct to the order of $\hbar^2 Q^2/M^2c^2$, is

$$\begin{aligned} H = -\frac{4\pi e^2}{q_\mu^2}\langle v_f| \sum_{j=1}^{4} \Bigg\{ & F_{1N}(q_\mu^2)\ e^{-i\mathbf{Q}\cdot\mathbf{x}_j} \\ & - \frac{F_{1N}(q_\mu^2)}{2M}[(\mathbf{p}_j\cdot\boldsymbol{\alpha})\ e^{-i\mathbf{Q}\cdot\mathbf{x}_j} + e^{-i\mathbf{Q}\cdot\mathbf{x}_j}\ (\mathbf{p}_j\cdot\boldsymbol{\alpha})] \\ & - i\left[\frac{F_{1N}(q_\mu^2)+\kappa F_{2N}(q_\mu^2)}{2M}\right]\boldsymbol{\sigma}_j\cdot(\mathbf{x}_j\times\boldsymbol{\alpha})\ e^{-i\mathbf{Q}\cdot\mathbf{x}_j} \\ & + \frac{q_\mu^2}{8M^2}\ [F_{1N}(q_\mu^2)+2\kappa F_{2N}(q_\mu^2)]\ e^{-i\mathbf{Q}\cdot\mathbf{x}_j}\Bigg\}|u_i\rangle\,. \end{aligned} \quad (4)$$

Here $\mathbf{x}_j$ and $\mathbf{p}_j$ are the position and momentum operators of the j-th nucleon, $\boldsymbol{\sigma}_j$ is the nucleon spin operator, $\boldsymbol{\alpha}$ is the Dirac matrix acting on the free electron spinors $|v_i\rangle$ and $|v_f\rangle$, while q_μ^2 is the exchanged four-momentum squared. F_{1N} and F_{2N} are the form factors of the nucleon, κ is the anomalous moment of the nucleon in nuclear magnetons, and M is the nucleon mass.

For proton knock-out, the transition matrix Eq. (3) reads

$$\mathcal{M} = -\langle v_f|v_i\rangle\mathcal{M}_Q + \langle v_f|\boldsymbol{\alpha}|v_i\rangle\cdot(\mathbf{M}_{\rm el}+\mathbf{M}_{\rm mag})\ , \quad (5)$$

where

$$\mathcal{M}_Q = 2^{(-)}\langle\mathbf{q};\Psi_{III}|\mathcal{H}_Q|\Psi_{IV}\rangle\,, \quad (6)$$

$$\mathbf{M}_{\rm el} = 2^{(-)}\langle\mathbf{q};\Psi_{III}|\mathbf{H}_{\rm el}|\Psi_{IV}\rangle\,, \quad (7)$$

$$\mathbf{M}_{\rm mag} = 2^{(-)}\langle\mathbf{q};\Psi_{III}|\mathbf{H}_{\rm mag}|\Psi_{IV}\rangle\,. \quad (8)$$

The Hamiltonians $\mathcal{H}_Q$, $\mathbf{H}_{\rm el}$, and $\mathbf{H}_{\rm mag}$ are given by

$$\mathcal{H}_Q = F_{\rm ch}^p(1+q_\mu^2/8M^2)\sum_{j=1}^{4} e^{-i\mathbf{Q}\cdot\mathbf{x}_j}\,\lambda_j\,, \quad (9)$$

$$\mathbf{H}_{\rm el} = (F_{\rm ch}^p/2M)\sum_{j=1}^{4}(\mathbf{p}_j e^{-i\mathbf{Q}\cdot\mathbf{x}_j} + e^{-i\mathbf{Q}\cdot\mathbf{x}_j}\mathbf{p}_j)\,\lambda_j\,, \quad (10)$$

$$\mathbf{H}_{\rm mag} = (i/2M)F_{\rm mag}^p\sum_{j=1}^{4} e^{-i\mathbf{Q}\cdot\mathbf{x}_j}\boldsymbol{\sigma}_j\times\mathbf{Q}\lambda_j\,, \quad (11)$$

Here the superscript p refers to the proton and $\lambda_j = (1+\tau_z^j)/2$ is the isospin operator for nucleon j while $F_{\rm ch}^p$ and $F_{\rm mag}^p$ are the charge and magnetic form factors of the proton defined by

$$F_{\rm ch}^p = F_{1p} + (q_\mu^2/4M^2)\kappa_p F_{2p} \quad (12)$$

$$F_{\rm mag}^p = F_{1p} + \kappa_p F_{2p}\,. \quad (13)$$

The analytical fit to the proton form factors F_{1p} and F_{2p} given by Janssens *et al.* [21] is used in the calculations.

Squaring the matrix element, summing and averaging over the electron spin, and inserting the resulting expression in Eq. (1), we obtain

$$\begin{aligned} \frac{d^5\sigma}{dE_f\,d\Omega_p\,d\Omega_e} = & \frac{\sigma_{\rm M}}{(\hbar c)^3(2\pi)^3}\ \frac{|\mathbf{p}_p|E_p}{1-\dfrac{E_p}{E_{^3H}}\dfrac{\mathbf{p}_p\cdot\mathbf{p}_{^3H}}{|\mathbf{p}_p|^2}} \\ & \Big\{|\mathcal{M}_Q|^2\ - \frac{1}{2}\sec^2\frac{\theta}{2}(\mathcal{M}_Q^*\mathbf{J}+\mathbf{J}^*\mathcal{M}_Q)\cdot(\hat{k}_i+\hat{k}_f) \\ & + \frac{1}{2}\sec^2\frac{\theta}{2}(\mathbf{J}\cdot\hat{k}_i\mathbf{J}^*\cdot\hat{k}_f + \mathbf{J}\cdot\hat{k}_f\mathbf{J}^*\cdot\hat{k}_i) \\ & + |\mathbf{J}|^2\tan^2\frac{\theta}{2}\Big\}\,, \end{aligned} \quad (14)$$

where $\mathbf{J} = \mathbf{M}_{\rm el}+\mathbf{M}_{\rm mag}$. The determination of the coincidence cross section is thus reduced to the determination of the nuclear matrix elements $\mathcal{M}_Q$ and $\mathbf{J}$.

In this work we use the dominant electric operators (9) and (10). In PWIA the nuclear matrix elements (6) and (7) read

$$\mathcal{B}_Q(\mathbf{q}) = 2\,\langle\mathbf{q}|\langle\Psi_{III}|\sum_{j=1}^{4}\exp(-i\mathbf{Q}\cdot\mathbf{x}_j)\,\lambda_j|\Psi_{IV}\rangle \quad (15)$$

and

$$\begin{aligned} \mathbf{B}_{\rm el}(\mathbf{q}) = 2\,\langle\mathbf{q}|\langle\Psi_{III}|\sum_{j=1}^{4}\ & (\mathbf{p}_j\exp(-i\mathbf{Q}\cdot\mathbf{x}_j) \\ & + \exp(-i\mathbf{Q}\cdot\mathbf{x}_j)\mathbf{p}_j)\ \lambda_j|\Psi_{IV}\rangle\ , \end{aligned} \quad (16)$$

where $\mathbf{q} = (\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3-3\mathbf{p}_4)/4$.

The operators appearing in Eqs. (15) and (16) are the same as those of Eqs. (9) and (10), except that the nucleonic form factors $F_{\rm ch}^p$ and $F_{\rm mag}^p$ are not noted here.

To proceed we express the operators $\mathbf{x}_j$ and $\mathbf{p}_j$ in Jacobi coordinates and neglect, as in the photodisintegration case [14,15], those acting within $|\Psi_{III}\rangle$. The remaining terms, containing $\mathbf{q}$ and its canonically conjugate counterpart, are treated without further approximation. A straightforward calculation then reduces (15) to

$$\begin{aligned} \mathcal{B}'_Q(\mathbf{q}) = & 2\,\langle\mathbf{q}+\frac{1}{4}\mathbf{Q}|\langle\Psi_{III}|(\lambda_1+\lambda_2+\lambda_3)|\Psi_{IV}\rangle \\ & + 2\,\langle\mathbf{q}-\frac{3}{4}\mathbf{Q}|\langle\Psi_{III}|\lambda_4|\Psi_{IV}\rangle, \end{aligned} \quad (17)$$

whereas (16) is replaced by

$$\begin{aligned} \mathbf{B}'_{\rm el}(\mathbf{q}) = & \frac{4}{3}\,\mathbf{q}\,\langle\mathbf{q}+\frac{1}{4}\mathbf{Q}|\langle\Psi_{III}|(\lambda_1+\lambda_2+\lambda_3)|\Psi_{IV}\rangle \\ & - 4\,\mathbf{q}\,\langle\mathbf{q}-\frac{3}{4}\mathbf{Q}|\langle\Psi_{III}|\lambda_4|\Psi_{IV}\rangle + \mathbf{Q}\,\mathcal{B}'_Q(\mathbf{q}). \end{aligned} \quad (18)$$

The construction of the above matrix elements requires the knowledge of the bound states $|\Psi_{III}\rangle$ and $|\Psi_{IV}\rangle$. For their calculation the exact three- and four-nucleon AGS

integral equations are employed [12,13]. The latter consist the coupled set of (before antisymmetrization) 18x18 four-body AGS equations. They contain in their kernel all subsystem information via the two-body T-matrices, the three- and (2+2)-body AGS transition operators. By this approach, the full coupling and the corresponding interference of (2+2)- and (3+1)-channels in the four-body system is taken into account explicitly, and thus exactly and completely. In order to reduce the original three- and four-body relations to (one-dimensional) integral equations, the W-matrix method [22] and the energy-dependent pole approximation (EDPA) [23] are used. In the purely nuclear case these approximations have led to very accurate results (see e.g. [24–26]). Furthermore, they have been successfully used in calculations of the photodisintegration of ^{3}H, ^{3}He [9,27] and ^{4}He [14,15,9]. The graphical representation of matrix elements like Eqs. (15) and (16), adapted to the four-body AGS formalism, can be found in [16]. As in [14,15], the Malfliet-Tjon potential I and III [28] is chosen, as it is both sufficiently realistic and simple enough to be employed in four-nucleon calculations. This property is of particular importance in our calculations where the computation of the matrix elements, despite the approximations used, is still tedious and of considerable numerical complexity. The corresponding binding energies are 8.595 MeV for ^{3}H and 30.1 MeV for ^{4}He [14].

The results obtained within the AGS formalism for the ^{4}He(e,e′p)^{3}H five-fold differential cross section as a function of the missing momentum $\mathbf{Q}$ are shown in Fig. 2. The kinematics and the experimental data are those of [4,5] for the $\omega = 215\,\mathrm{MeV}$ case. For comparison we also included in the figure the PWIA results of [8], obtained for wave functions constructed via the integrodifferential equation approach (IDEA) of Ref. [29]. The PWIA results of Laget (see Refs. [4,5,7]) are also shown, being obtained for the Urbana potential and for wave functions constructed with the variational Monte Carlo (MC) method. The agreement of our AGS calculations with the experimental data, especially in the region where the PWIA results of the other two methods show their characteristic dip, is remarkable. This holds true also in comparison with the other results reported in [5]. Fig. 3 shows the five-fold differential cross section for the ^{4}He(e,e′p)^{3}H reaction for the Saclay kinematics [3]. For comparison the Laget results [7] are also plotted. The agreement of our results with experiment is again remarkable.

The overall small discrepancies may be reduced by using a better NN force, further improvements in the PWIA matrix elements, and inclusion of the FSI in a rigorous way. A particular advantage of the AGS-type approach in this respect lies in the fact that the incorporation of the FSI is exactly the same for the four-nucleon scattering, the photodisintegration of ^{4}He, and the electrodisintegration of ^{4}He (see e.g. [9] and Refs. therein). Most important: in this approach the underlying integral equations explicitly incorporate the (2+2)-channels, not fully included in other approaches, and their coupling to the (3+1)-channels. That underlying interference of competing channels is the most relevant feature of four-body theory as compared to three-body theory. In other words, the complexity of four-body rearrangement processes is fully taken into account.

In conclusion, our results show that already in PWIA quite a good description of the experimental data can be achieved. The main reason for this agreement appears to be the use of wave functions obtained from the AGS integral equations with their complete coupling scheme. Another reason is the way of calculating the nuclear matrix elements. Namely, those parts of the electromagnetic operators (9) and (10), which act between the relative motion of the two outgoing nuclear fragments, are taken into account exactly. The sensitivity to the input NN-potential and to the above-mentioned 2+2 rearrangement terms are under investigation.

ACKNOWLEDGMENTS

Financial support from the University of South Africa and the Foundation for Research Development of South Africa is appreciated.

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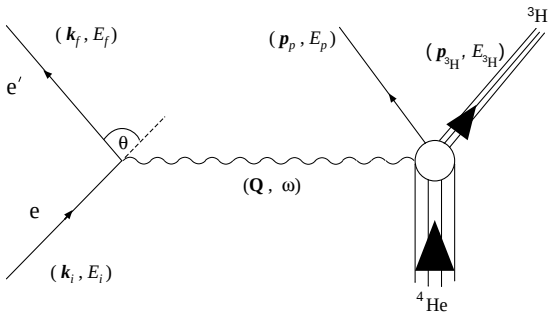

FIG. 1. Kinematics for the process ^{4}He(e, e′p)^{3}H.

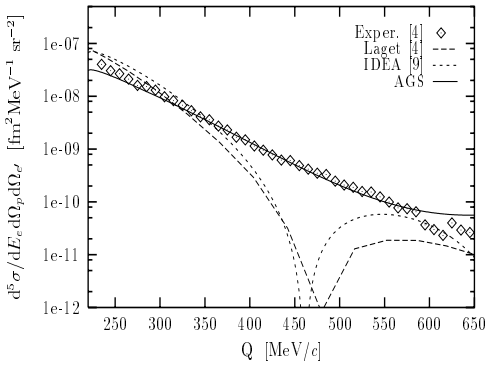

FIG. 2. Five-fold ^{4}He(e,e′ p)^{3}H differential cross section as a function of the missing momentum Q for the NIKHEF kinematics [4,5].

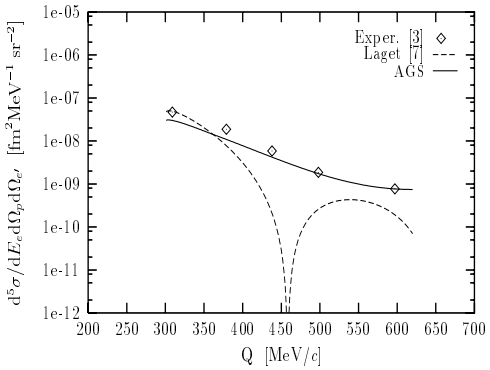

FIG. 3. Five-fold ^{4}He(e,e′p)^{3}H differential cross section as a function of the missing momentum Q for the the Saclay kinematics [3].